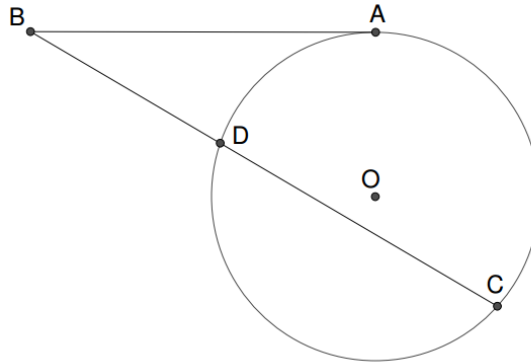


**Caltech Harvey Mudd
Mathematics Competition**

Tiebreaker Round

November 23, 2013

1. In the diagram below, point A lies on the circle centered at O . AB is tangent to circle O with $\overline{AB} = 6$. Point C is $\frac{2\pi}{3}$ radians away from point A on the circle, with BC intersecting circle O at point D . The length of BD is 3. Compute the radius of the circle.



2. Suppose the roots of

$$x^4 - 3x^2 + 6x - 12 = 1$$

are α , β , γ , and δ . What is the value of

$$\frac{\alpha + \beta + \gamma}{\delta^2} + \frac{\alpha + \delta + \gamma}{\beta^2} + \frac{\alpha + \beta + \delta}{\gamma^2} + \frac{\delta + \beta + \gamma}{\alpha^2} ?$$

3. Bill plays a game in which he rolls two fair standard six-sided dice with sides labeled one through six. He wins if the number on one of the dice is three times the number on the other die. If Bill plays this game three times, compute the probability that he wins at least once.

4. Let

$$A = \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{9},$$

$$B = \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 5} + \frac{1}{2 \cdot 9} + \frac{1}{3 \cdot 5} + \frac{1}{3 \cdot 9} + \frac{1}{5 \cdot 9},$$

$$C = \frac{1}{2 \cdot 3 \cdot 5} + \frac{1}{2 \cdot 3 \cdot 9} + \frac{1}{2 \cdot 5 \cdot 9} + \frac{1}{3 \cdot 5 \cdot 9}.$$

Compute the value of $A + B + C$.