

Tiebreaker Round Solutions

2014 CHMMC

Solution 1.

$$\boxed{\frac{65}{72}}$$

Solution 2. We have $y = b(a + d)$ and $z = c(a + d)$, so $b = y/(a + d)$ and $c = z/(a + d)$. Thus b and c are entirely determined by a and d . Plugging these in gives $a^2 + bc = a^2 + yz(a + d)^2$ and $d^2 + bc = d^2 + yz(a + d)^2$. The equations $a^2 + yz(a + d)^2 = x$ and $d^2 + yz(a + d)^2 = w$ clearly have two roots with respect to a and d since $x, w \neq 0$. Therefore given a valid a , there are two possible values of d , and given a valid d there are two possible values of a . This gives a total of $\boxed{4}$ possibilities. Alternatively, use diagonalization; by solving the roots of the characteristic polynomial we get $\lambda = 3 \pm \sqrt{7}$:

$$\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} = P \begin{bmatrix} 3 + \sqrt{7} & 0 \\ 0 & 3 - \sqrt{7} \end{bmatrix} P^{-1}$$

which must give the roots

$$\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} = P \begin{bmatrix} \pm\sqrt{3 + \sqrt{7}} & 0 \\ 0 & \pm\sqrt{3 - \sqrt{7}} \end{bmatrix} P^{-1}$$

where the \pm are independent. This again gives 4 possibilities.

Solution 3. If you're not in one of these positions, you can move to one of them: 1, 3, 6 removes 1 bean, 4 removes 4 beans and 7 removes 7 beans.

If you are in one of these positions, you can't move to one of them: $0 \mapsto \{6, 4, 1\}$; $2 \mapsto \{1, 6, 3\}$; $5 \mapsto \{4, 1, 6\}$

The winning strategy for this game is to always move to $n \equiv 0, 2, 5 \pmod{8}$ if possible, so player 2 wins whenever $n \equiv 0, 2, 5 \pmod{8}$. There are $\boxed{14}$ numbers between 2014 and 2050 satisfying this.

Solution 4. $f(n, 0, 0) = n^2$.

The second coordinate keeps a running total of the sum of the various values of the third coordinate as the first coordinate goes to 0. In other words, $f(n, 0, 0) = \sum_{i=1}^n (2i - 1) = \boxed{n^2}$

Solution 5. Let the expression be x . Take the natural log to give

$$\ln x = \sum_{n=1}^{\infty} \frac{n}{3^n} \ln 3$$

and the sum reduces to

$$\begin{aligned} \ln x &= \frac{1}{3} \ln 3(3/2)^2 \\ x &= \boxed{3^{3/4}} \end{aligned}$$